1.6 CHAPTER SUMMARY

This chapter introduces the concepts of signals and systems. Both signals and systems are broken into the categories of continuous time and discrete time. Continuous-time signals are signals $x(t)$ in which time t assumes values from the set of real numbers, while discrete-time signals $x[n]$ are defined only at integer values of the index n. Continuous-time signals arise naturally, with common examples being voltages and currents in circuits, velocities and positions of moving objects, flow rates and pressures in chemical processes, and speech and ECGs in humans. Discrete-time signals are often obtained by sampling a continuous-time signal, that is, defining the signal only at discrete points in time $t = nT$, where T is the sampling time. Discrete-time signals also arise "naturally," as shown in the example given in this chapter involving the closing price of a particular stock at the end of each trading day.

There are several common signals defined in this chapter that will be used throughout the text. The names are the same for the continuous-time and discrete-time versions of these signals: step function, ramp function, and sinusoidal function. In addition, there is the unit-impulse function for continuous-time signals and the unit-pulse function for discrete-time signals. Impulse and pulse functions are very important for analyzing engineering systems. It will be shown in Chapter 2 that the response of the system to the unit impulse or unit pulse is a means of characterizing the system. The discrete-time step function, ramp function, and sinusoidal function can be viewed as sampled versions of their continuous-time versions, but a scaled version of the sampled ramp function is more often used, $r[n] = nu[n]$. It is interesting to note that a continuous-time sinusoid $\cos(\omega t + \theta)$ is periodic, but a discrete-time sinusoid $\cos(\Omega n + \theta)$ is periodic if and only if there exists a positive integer r such that $\Omega = 2\pi q/r$ for some integer q.

A system is a collection of one or more devices or processes that operates on an input signal to produce an output signal. Examples of systems include electrical circuits, communication systems, signal processors, biological systems, robotic manipulators, chemical processes, and manufacturing systems. There are several representations of systems; some of them are considered as time domain, and others are considered as frequency domain models. Time domain representations include the convolution model and the input/output differential, or difference equation, to be studied in Chapter 2. The frequency domain models include the Fourier transform, to be studied in Chapters 3–5, and the transform models to be studied in Chapters 6–8.

There are several important properties of continuous-time and discrete-time systems, including linearity, time invariance, causality, and memory. Several powerful analysis methods exist, such as the Fourier transform and transfer function approaches, that can be used to study the behavior of systems which have these properties. The remaining chapters in the text focus on these analysis methods.

PROBLEMS

1.1. Consider the continuous-time signals displayed in Figure P1.1.

(i) Show that each of these signals is equal to a sum of rectangular pulses $p_\tau(t)$ and/or triangular pulses $(1 - 2|t|/\tau)p_\tau(t)$.

(ii) Use MATLAB to plot the signals in Figure P1.1.

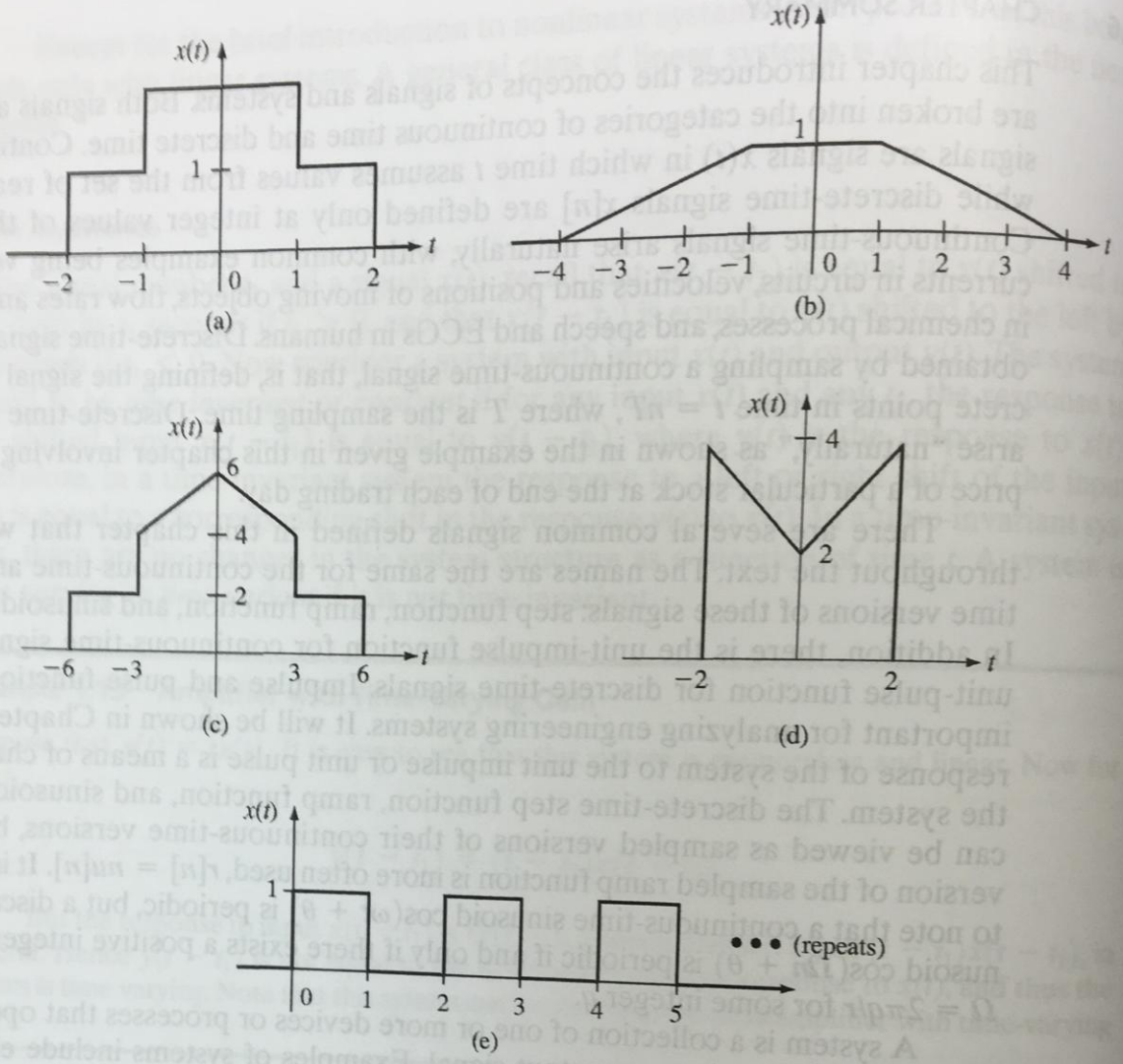


FIGURE P1.1

1.2. Obtaining a computer-generated plot of a continuous-time signal requires some care in choosing the time increment Δt (the spacing between points). As mentioned in Section 1.1, too large of an increment will cause a jagged plot. Moreover, a very large time increment may introduce a phenomenon known as aliasing, which distorts the information given in the signal. (Aliasing is covered in greater detail in Chapter 5.) To avoid aliasing when defining a computer-generated sinusoid such as $x(t) = \cos(\omega t + b)$, choose $\Delta t \leq \pi/\omega$. A rule of thumb in the case of a decaying sinusoid such as $x(t) = e^{-at}\cos(\omega t + b)$ is to choose $\Delta t \leq \pi/\left(4\sqrt{a^2 + \omega^2}\right)$. (Choosing even smaller values of Δt creates smoother plots.)

(a) Compute the maximum time increment for plotting $x(t) = \sin \pi t$ in MATLAB. Verify your result by plotting $x(t)$ for $t = 0$ to $t = 20$ sec with the following time increments: $\Delta t = 0.1$ sec, $\Delta t = 0.5$ sec, $\Delta t = 0.9$ sec, $\Delta t = 1.5$ sec. Note the apparent change in frequency in the plot due to the aliasing effect for $\Delta t = 1.5$. How do you expect your plot to appear when $\Delta t = 2$ sec? Verify your result.

(b) Compute the maximum time increment for plotting $x(t) = e^{-0.1t}\cos \pi t$. Verify your result by plotting $x(t)$ for $t = 0$ to $t = 20$ sec with $\Delta t = 0.1$, 0.5, 1.5, and 2 sec.

(c) Compute the maximum time increment for plotting $x(t) = e^{-t}\cos(\pi t/4)$. Verify your result by plotting $x(t)$ for $t = 0$ to $t = 10$ sec with $\Delta t = 0.1, 1, 2,$ and 3 sec.

The problem with aliasing exists not only with plotting, but with all digital processing of continuous-time signals. A computer program that emulates a continuous-time system needs to have an input signal defined so that the signal has very little aliasing.

1.3. Use MATLAB to plot the functions (a)–(h) for $-1 \le t \le 5$ sec. Label your axes appropriately.

(a) the unit-step function $u(t)$

(b) the unit-ramp function $r(t)$

(c) $x(t) = \begin{cases} 1, & 0 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$

(d) $x(t) = 10e^{-3t}\,u(t)$

(e) $x(t) = 3e^{-t}\cos 2t\,u(t)$

(f) $x(t) = 3e^{t}\cos 2t\,u(t)$

(g) $x(t) = 2\sin(3t - \pi/2) - \cos 2t$

(h) $x(t) = \sin 5t + \sin \pi t$

1.4. Sketch the continuous-time signals in parts (a) to (c).

(a) $x(t) = u(t + 1) - 2u(t - 1) + u(t - 3)$

(b) $x(t) = (t + 1)u(t - 1) - tu(t) - u(t - 2)$

(c) $x(t) = 2(t - 1)u(t - 1) - 2(t - 2)u(t - 2) + 2(t - 3)u(t - 3)$

(d) Use MATLAB to plot the signals defined in parts (a) to (c).

1.5. Use MATLAB to plot the following signals over the range $-1 \le t \le 5$:

(a) $x(t) = e^{-t}\,u(t) + e^{-t}[\exp(2t - 4) - 1]u(t - 2) - e^{t-4}u(t - 4)$

(b) $x(t) = \cos t\left[u\left(t + \frac{\pi}{2}\right) - 2u(t - \pi)\right] + (\cos t)u\left(t - \frac{3\pi}{2}\right)$

1.6. Given a continuous-time signal $x(t)$ and a constant c, consider the signal $x(t)u(t - c)$.

(a) Show that there exists a signal $v(t)$ such that

$$x(t)u(t - c) = v(t - c)u(t - c)$$

Express $v(t)$ in terms of $x(t)$.

(b) Determine the simplest possible analytical form for $v(t)$ when

(i) $x(t) = e^{-2t}$ and $c = 3$

(ii) $x(t) = t^2 - t + 1$ and $c = 2$

(iii) $x(t) = \sin 2t$ and $c = \pi/4$

1.7. Plot the following discrete-time signals:

(a) $x[n]$ = discrete-time unit-step function $u[n]$

(b) $x[n]$ = discrete-time unit-ramp function $r[n]$

(c) $x[n] = (0.5)^n\,u[n]$

(d) $x[n] = (-0.5)^n\,u[n]$

(e) $x[n] = 2^n\,u[n]$

(f) $x[n] = \sin(\pi n/4)$

(g) $x[n] = \sin(\pi n/2)$

(h) $x[n] = (0.9)^n[\sin(\pi n/4) + \cos(\pi n/4)]$

(i) $x[n] = \begin{cases} 1, & -4 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$

(j) Verify the plots for parts (a)–(i) by using MATLAB with the `stem` command `stem(n,x,'filled')`. Label your axes appropriately.

1.8. Sketch the following discrete-time signals:

(a) $x[n] = u[n] - 2u[n-1] + u[n-4]$

(b) $x[n] = (n+2)u[n+2] - 2u[n] - nu[n-4]$

(c) $x[n] = \delta[n+1] - \delta[n] + u[n+1] - u[n-2]$

(d) $x[n] = e^{0.8n}u[n+1] + u[n]$

(e) Use MATLAB to plot the signals defined in parts (a) to (d).

1.9. Use an analytical method to determine if the signals (a)–(f) are periodic; if so, find the fundamental period. Use MATLAB to plot each signal, and verify your prediction of periodicity. Use a small enough time increment for the continuous-time signals to make your plot smooth (see Problem 1.2).

(a) $x(t) = \cos \pi t + \cos(4\pi t/5)$

(b) $x(t) = \cos(2\pi(t-4)) + \sin 5\pi t$

(c) $x(t) = \cos 2\pi t + \sin 10t$

(d) $x[n] = \sin 10n$

(e) $x[n] = \sin(10\,\pi n/3)$

(f) $x[n] = \cos(\pi n^2)$ (*Hint:* To get n^2 when n is stored in a vector, type n.^2.)

1.10. For the N-point MA filter given by the input/output relationship (1.16), where N is any positive integer, derive a mathematical expression for the filter's output response $y[n]$ when the input $x[n]$ is

(a) the unit-pulse function $\delta[n]$.

(b) the discrete-time unit-step function $u[n]$.

(c) the discrete-time unit-ramp function $r[n]$.

1.11. In Problem 1.10c, determine the time delay in the N-point MA filter response $y[n]$ when $n > N$.

1.12. Write a MATLAB M-file to compute the output response $y[n]$ of the 6-point MA filter to the input $x[n] = 5\sin(\pi n/10 + \pi/4)$, for $0 \le n \le 69$. Express your answer by giving the M-file and the MATLAB plot.

1.13. For your solution in Problem 1.12, determine the time delay in the filter response.

1.14. Consider the system given by the input/output relationship

$$y[n] = \frac{32}{63}\left[x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2] + \frac{1}{8}x[n-3] + \frac{1}{16}x[n-4] + \frac{1}{32}x[n-5]\right]$$

(a) When $x[n]$ is equal to a constant c for all $n \ge 0$, show that $y[n]$ is equal to c for all $n \ge 5$.

(b) Write a MATLAB M-file to compute the output response $y[n]$ resulting from the input $x[n] = 5\sin(\pi n/10 + \pi/4)$, for $0 \le n \le 69$. Express your answer by giving the M-file and the MATLAB plot.

(c) Compare your result in part (b) with the MA filter response obtained in Problem 1.12.

1.15. The continuous-time counterpart to the N-point discrete-time MA filter is the I-interval continuous-time MA filter given by the input/output relationship

$$y(t) = \frac{1}{I}\int_{t-I}^{t} x(\lambda)\, d\lambda$$

where I is a positive number (the time interval over which the input is integrated). Derive a mathematical expression for the filter's output response $y(t)$ when the input $x(t)$ is

(a) the unit impulse $\delta(t)$.

(b) the continuous-time unit-step function $u(t)$.

(c) the continuous-time unit-ramp function $r(t)$.

(d) For your result in part c, determine the delay in the filter's response when $t > I$.

1.16. Using the procedure given in Section 1.2, download from the Web the closing prices of QQQQ for the 60-business-day period April 27, 2004, through July 22, 2004. Give the MATLAB plot for $9 \leq n \leq 60$ for the closing prices and the nine-day MA filter response to the data.

1.17. Determine whether these continuous-time systems are causal or noncausal, have memory or are memoryless. Justify your answers. In the following parts, $x(t)$ is an arbitrary input and $y(t)$ is the output response to $x(t)$:

(a) $y(t) = |x(t)| = \begin{cases} x(t) & \text{when } x(t) \geq 0 \\ -x(t) & \text{when } x(t) < 0 \end{cases}$

(b) $y(t) = e^{x(t)}$

(c) $y(t) = (\sin t)x(t)$

(d) $y(t) = \begin{cases} x(t) & \text{when } |x(t)| \leq 10 \\ 10 & \text{when } |x(t)| > 10 \end{cases}$

(e) $y(t) = \displaystyle\int_0^t (t - \lambda)x(\lambda)\, d\lambda$

(f) $y(t) = \displaystyle\int_0^t \lambda x(\lambda)\, d\lambda$

1.18. Prove that the integral system and the differential system given next are both linear:

(i) $y(t) = \displaystyle\int_0^t x(\lambda)\, d\lambda$

(ii) $y(t) = \dfrac{dx(t)}{dt}$

1.19. For each of the systems in Problem 1.17, determine whether the system is linear or nonlinear. Justify your answers.

1.20. For each of the systems in Problem 1.17, determine whether the system is time invariant or time varying. Justify your answers.

1.21. A continuous-time system is said to have a *dead zone* if the output response $y(t)$ is zero for any input $x(t)$ with $|x(t)| < A$, where A is a constant called the *threshold*. An example is a dc motor that is unable to supply any torque until the input voltage exceeds a threshold value. Show that any system with a dead zone is nonlinear.

1.22. Determine whether the circuit with the ideal diode in Figure P1.22 is causal or noncausal, linear or nonlinear, and time invariant or time varying. Justify your answers.

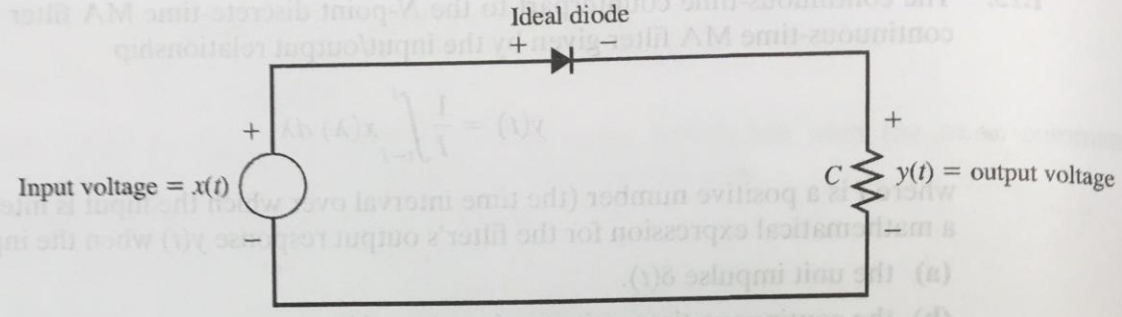


FIGURE P1.22

1.23. Suppose that $x(t)$ is the input to a linear time-invariant system with corresponding output $y(t)$. Prove that the system has the following properties:

(a) An input of $x_1(t) = \int_0^t x(\lambda)\, d\lambda$ to the same system yields an output of $y_1(t) = \int_0^t y(\lambda)\, d\lambda$.

(b) An input of $x_1(t) = \dfrac{dx(t)}{dt}$ to the same system yields an output of $y_1(t) = \dfrac{dy(t)}{dt}$.

1.24. A linear time-invariant continuous-time system responds to the following inputs with the corresponding outputs: If $x(t) = u(t)$, then $y(t) = 2(1 - e^{-t})\, u(t)$ and if $x(t) = \cos t$, then $y(t) = 1.414 \cos(t - \pi/4)$. Find $y(t)$ for the following inputs (Hint: Use Problem 1.23 for part (d)):

(a) $x(t) = 2u(t) - 2u(t - 1)$

(b) $x(t) = 4 \cos(2(t - 2))$

(c) $x(t) = 5u(t) + 10 \cos(2t)$

(d) $x(t) = tu(t)$

1.25. To understand better the concept of linearity in discrete-time systems, write a MATLAB M-file that generates the output response $y[n]$ for $0 \le n \le 30$ of the five-day MA filter to the following various inputs:

(a) Compute and give the MATLAB plot of the response $y_1[n]$ to the input $x[n] = u[n]$.

(b) Compute and give the MATLAB plot of the response $y_2[n]$ to the input $x[n] = 2u[n]$. Compare this response with that obtained in part (a).

(c) Compute and give the MATLAB plot of the response $y_3[n]$ to the input $x[n] = \sin(\pi n/4)\, u[n]$.

(d) Compute and give the MATLAB plot of the response $y_4[n]$ to the input $x[n] = 2u[n] + \sin(\pi n/4)\, u[n]$. Compare this response with $2y_1[n] + y_3[n]$.

1.26. To understand better the concept of time-invariance in discrete-time systems, write a MATLAB M-file that generates the output response $y[n]$ for $0 \le n \le 30$ to the following various inputs for the system defined in Problem 1.14:

(a) Compute and give the MATLAB plot of the response $y_1[n]$ to the input $x[n] = u[n]$.

(b) Compute and give the MATLAB plot of the response $y_2[n]$ to the input $x[n] = u[n - 2]$. Compare this response with that obtained in part (a).

(c) Compute and give the MATLAB plot of the response $y_3[n]$ to the input $x[n] = \sin(\pi n/4)\, u[n]$.

(d) Compute and give the MATLAB plot of the response $y_4[n]$ to the input $x[n] = \sin(\pi[n - 4]/4)u[n - 4]$. Compare this response with $y_3[n - 4]$.

1.27. Prove that the following system is linear:

$$y[n] = \sum_{i=0}^{n} a_i x[n - i]$$

where the coefficients a_i are constants.

1.28. Determine whether the system defined in Problem 1.14 is causal or noncausal, linear or nonlinear, and time invariant or time varying. Justify your answers.

1.29. Determine whether the following discrete-time systems are causal or noncausal, have memory or are memoryless, are linear or nonlinear, are time invariant or time varying. Justify your answers. In the following parts, $x[n]$ is an arbitrary input and $y[n]$ is the response to $x[n]$.

(a) $y[n] = x[n] + 2x[n - 2]$

(b) $y[n] = x[n] + 2x[n + 1]$

(c) $y[n] = nx[n]$

(d) $y[n] = u[n]x[n]$

(e) $y[n] = |x[n]|$

(f) $y[n] = \sin x[n]$

(g) $y[n] = \sum_{i=0}^{n} (0.5)^n x[i],\ n \geq 0$